Seat No. ________________

HP-003-1162003

M. Sc. (Sem. II) Examination

April - 2023

Mathematics : CMT - 2003

(Topology - II)

Faculty Code : 003

Subject Code : 1162003

Time : $2\frac{1}{2}$ Hours / Total Marks : **70**

Instructions : (1) All questions are compulsory.
(2) Each question carries equal marks.
(3) Figure on the right indicate alloted marks.

1 Answer any **seven** short questions : **7×2=14**

(1) Define term : T_2 - Space and give an example of T_2-Space.

(2) Let $f : X \to Y$ is onto, continuous, closed and X is a T_1-Space. Prove that, Y is a T_1-Space.

(3) Give an example of a space which is not compact.

(4) Prove that, $\mathbb{R}$ with standard topology is a T_2-Space.

(5) Prove that, if Y is an open subspace of a locally compact, Hausdorff space X then Y is locally compact.

(6) Prove that, every compact Hausdorff space is regular.

(7) Prove that, every subspace of a completely regular space is completely regular.

(8) Define terms : Cover and Subcover.

(9) Prove that, Every Cauchy sequence (x_n) in $(\mathbb{R}, d)$ is a bounded sequence.

(10) Define term : Completion in a metric space and give an example of it.

2 Attempt any **two** : **2×7=14**

(1) If X is a T_2-Space and (x_n) is a sequence in X and suppose $(x_n) \to x$ as well as $(x_n) \to y$ in X. Prove that, $x = y$.

(2) Prove that, a space X is a T_1-Space if and only if for every $x \in X, \{x\}$ is a closed subspace of X.

(3) Prove that, a space X is a Hausdorff if and only if the set $\Delta = \{(x, x) / x \in X\}$ is a closed subset of $X \times X$.

3 Attempt followings : **2×7=14**

(1) Prove that, $X \times Y$ is compact if and only if both X and Y are compacts.

(2) Prove that, if X is compact Hausdorff space without isolated points then X must be uncountable.

OR

3 Attempt followings : **2×7=14**

(1) Prove that, a subspace Y of $\mathbb{R}$ is compact if and only if Y is closed and bounded subset of $\mathbb{R}$.

(2) State and prove, Lebesgue Covering Lemma.

4 Attempt any **two** : **2×7=14**

(1) Let $f : X \to Y$ is onto, open and continuous and X is locally compact. Prove that, Y is also locally compact.

(2) Prove that, $X \times Y$ is locally compact if and only if X and Y are locally compact spaces.

(3) If X and Y are homeomorphic and X is regular then prove that Y is also regular.

5 Attempt any **two** : **7×2=14**

(1) Prove that, $X \times Y$ is completely regular if and only if X and Y are completely regular.

(2) Prove that, a T_1-Space X is normal iff whenever $A \subset X$ is closed, $U \subset X$ is open and $A \subset U$, there is an open set V such that $A \subset V \subset \overline{V} \subset U$.

(3) Prove that, a compact, Hausdorff space is normal.

(4) Let (X, d) be a metric space and (x_n) be a sequence in X. Prove that (x_n) is Cauchy in (X, d) if and only if (x_n) is Cauchy in $(X, \overline{d})$.